Phys 402
Fall 2022
Homework 11
Due Wednesday, 7 December @ 10 AM

FINAL EXAM will be on Tuesday 20 December (8:00 to 10:00 AM) in Toll/Physics 1402, covering everything

1. Griffiths, 3rd Edition, Problem 5.20 **[Average energy of electrons in a free-electron Fermi gas]**

2. Griffiths, 3rd Edition, Problem 5.21 **[Fermi gas properties of Copper]**

3. Griffiths, 3rd Edition, Problem 5.23 **[Bulk modulus of a free-electron Fermi gas]**

4. Griffiths, 3rd Edition, Problem 5.30. **[Fermi energy for 2D electrons]**

5. Griffiths, 3rd Edition, Problem 5.32. **[Electron gas with spin]**

6. Griffiths, 3rd Edition, Problem 5.33. **[Pauli paramagnetism]**

Extra Credit

11. Bose-Einstein Condensation

(a) Show that for bosons the chemical potential must always be less than the minimum allowed energy, which we take to be 0. *Hint*: the number of bose particles with energy E is $n(E) = \frac{1}{e^{(E-\mu)/k_BT}-1}$, and cannot be negative.

(b) In particular, for the ideal bose gas, $\mu(T) < 0$ for all T. Show that in this case $\mu(T)$ monotonically increases as T decreases, assuming the total number of particles N and the volume V are held constant. *Hint*: Study this expression for the total number of particles: $N = \frac{V}{2\pi^2}\int_0^\infty \frac{k^2\ dk}{e^{[(\hbar^2k^2/2m)-\mu]/k_BT}-1}$.

(c) A crisis (called Bose condensation) occurs when (as we lower T) $\mu(T)$ hits zero. Evaluate the integral, for $\mu = 0$, and obtain the formula for the critical temperature T_c at which this happens. Below the critical temperature, the particles crowd into the ground state, and the calculational device of replacing the discrete sum ($N = \sum_{n=1}^{\infty} N_n$, where N_n is the number of occupied states in energy level E_n) by a continuous integral (the expression for N given in (b)) loses its validity. Hint:

$$\int_0^\infty \frac{x^{s-1}}{e^x-1}\,dx = \Gamma(s)\,\zeta(s),$$

where Γ is Euler's gamma function, and ζ is the Reimann zeta function.

(d) Find the critical temperature for ^{4}He. Its density, at this temperature, is 0.15 gm/cm^3. *Comment*: The experimental value of the critical temperature in ^{4}He is 2.17 K.